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Event weights for simulating Bose-Einstein correlations

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Abstract

An event weighting method for simulating Bose-Einstein effects in hadronic final states is presented. The weight for an event depends on the momentum distribution of identical bosons in the event. By using a theoretically motivated parametrisation allowing weights below as well as above unity, the necessity of a weight-rescaling procedure is eliminated. A single parameter is used to adjust the average event weight to unity. Once adjusted, the same value of the parameter gives average event weights that are essentially independent of energy, initial quark flavour, multiplicity and jet topology. The influence of Bose-Einstein correlations on various measurable quantities in W pair production is found to be small. In particular, none of the scenarios considered resulted in a W mass shift larger than 20 MeV.

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1 Introduction

There are two main reasons for the renewed interest in Bose-Einstein correlations (BEC) in particle physics. One is the quark-gluon plasma search in high energy heavy ion collisions, where BEC are providing important information about the space-time development of the final hadron formation in the dense matter. The other is connected to the precision measurement of the W mass in e^+e^- annihilation, which can be used to constrain the allowed range of the Higgs boson mass in the Standard Model, or restrict the parameter space of any other "new physics". However, it was suggested that in the fully hadronic channel, $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$, BEC and colour reconnection effects could lead to significant uncertainties in the determination of W mass, up to $\mathcal{O}(100 \text{ MeV})$ [1,2], which can effectively render this channel useless and significantly reduce the precision on the W mass achievable at LEP2.

Existing Monte Carlo simulation programs for hadronic final states are based on the factorisation property of the QCD amplitudes [3]: the cross sections are defined by the perturbative parton level amplitudes, while the hadronisation process of the final quark states is simulated in the framework of a particular model, assuming that it does not change the probability of the perturbative part. Various parameters of the hadronisation models have been finely tuned to reproduce many aspects of the data, with a notable exception of Bose-Einstein correlations, which cannot be simulated in this approach in principle.

Several attempts have been made to implement Bose-Einstein effects a posteriori, so that the characteristic BEC are reproduced without breaking down the good description of other aspects of the data. At the moment, the most popular approach is the one developed in [2] and implemented in the PYTHIA Monte Carlo generator [4]. This model is based on the assumption that the Bose-Einstein effects are local in phase space and are introduced as shifts in final-state boson momenta (hence the rather misleading name of "local reweighting"). The advantage of this method is that the QCD factorisation is explicitly preserved, and, thus, cross-sections of the processes are not affected. This procedure, however, does not conserve energy. In an early approach, energy conservation was restored by rescaling all final-state hadron momenta, while in later algorithms the energies were corrected locally. In this model, a W mass shift arises as a consequence of the final-state boson momentum re-distribution. Numerical predictions depend on the details of the rescaling procedure and vary from 0 up to 100 MeV. The width of the W boson is also increased by up to 40 MeV. A major disadvantage of this method is that particle momentum re-distribution necessitates the re-tuning of the hadronisation parameters in order to reproduce the data, which makes the BEC effects rather difficult to extract.

Another approach to the implementation of Bose-Einstein effects is the event weighting method (referred to as "global reweighting" in [2]). Here, BEC are introduced by assigning weights to the events according to the momentum distributions of identical hadrons in the final state. Within certain simplifying assumptions, this procedure can be justified using the formalism of Wigner functions [5]. A number of such algorithms have been used recently to study the Bose-Einstein effects in the reaction $e^+e^- \to W^+W^-$ [6–10] (see also [2,11] for the comparative analysis of different methods). Although the various methods differ significantly in the prescriptions for weight calculation, one thing in common to all of them is a relatively small BEC-induced shift in the W mass, less than about 20 MeV. Apart from being much more appealing theoretically, event weighting has also another advantage compared to "local reweighting": it can, in principle, be applied to the existing Monte Carlo samples.

However, the event weighting methods also have some serious shortcomings. The distribution of event weights is usually very broad (if not divergent); average weights, if taken literally, are usually much larger than unity, and in order to keep the cross sections intact a rather arbitrary procedure of weight rescaling is used. Average weights may also vary for various event

classes such as different initial quark flavours, number of jets in the event, multiplicity etc. Thus, factorisation is not guaranteed, and is usually preserved by applying an *ad hoc* weight rescaling procedure for each class of events separately.

These difficulties can be traced back to the fact that event weights were larger than unity by construction, implying that BEC enhance configurations where identical bosons are close to each other in the phase space. However, Bose symmetry can generate repellent forces too, which may become dominant in some areas of the phase space (e. g. identical pions may not exist in a P-wave, so the decay $\rho \to \pi^0 \pi^0$ is forbidden), and give rise to event weights below unity. We use a theoretically motivated parametrisation which allows some event weights to fall below unity, and thus avoid the necessity of weight rescaling. In our method the average event weight in e⁺e⁻annihilation events is adjusted to unity using a single parameter, which appears to be independent of energy, initial quark flavour, number of jets or particle multiplicity in the event. Inclusive spectra of various hadrons also remain unaffected by the weighting procedure.

The theoretical motivation and description of our method is presented in the following section. The choice of the model parameters is discussed in Section 3. In Section 4 the method is applied to e⁺e⁻annihilation into hadrons in the energy range from 30 to 200 GeV, with the region around the Z peak studied in detail. The influence of Bose-Einstein effects on the process of W pair production is analysed in Section 5. Some conclusion are drawn in Section 6.

2 Motivation and algorithm description

Let M be the matrix element describing the production of a hadronic final state which, among other, non-identical particles, contains n identical bosons. This amplitude consists of n! terms, each corresponding to a particular permutation P of the n identical particles in the final state:

$$M = \sum_{P} M_{P} . (1)$$

When this process is simulated, the probabilistic treatment of the hadronisation stage means that the interference between different amplitudes is not included in the simulation:

$$|M|_{MC}^2 = \sum_P |M_P|^2 \neq |M|^2.$$
 (2)

As shown in [12], in order to take interference terms into account and thus restore the correct symmetry properties of the process, a weight w_P has to be assigned to each event:

$$|M|^2 = \sum_P w_P |M_P|^2, (3)$$

where

$$w_{P} = \sum_{P'} \frac{2\operatorname{Re}(M_{P}M_{P'}^{*})}{|M_{P}|^{2} + |M_{P'}|^{2}}$$

$$= 1 + \sum_{P' \neq P} \frac{2\operatorname{Re}(M_{P}M_{P'}^{*})}{|M_{P}|^{2} + |M_{P'}|^{2}}.$$
(4)

The sum contains n! terms and depends on the kinematical properties of the event. However, in order to be useful, the above formula needs to be implemented in a recipe for weight calculation.

Consider a simple parametrisation for the matrix element M_P , based on the Lund model of string hadronisation [12, 13]:

$$M_P = \exp[(i\kappa - b/2)A_P] . (5)$$

Here A_P stands for the integral over the space-time area of the string fragmentation, while κ and b are constants describing string tension and its breaking probability, respectively. Substituting (5) into (4), one obtains:

$$w_P = 1 + \sum_{P' \neq P} \frac{\cos(\kappa \Delta A_{PP'})}{\cosh(\frac{b}{2} \Delta A_{PP'})} , \qquad (6)$$

where $\Delta A_{PP'} = A_P - A_{P'}$.

In [12] it was argued that the dimensionless combination $\kappa \Delta A_{PP'}$ between the two configurations labelled P and P' can be estimated as the scalar product of the differences in 4-momentum and in the space-time. The event weights were calculated at the stage of event generation by the JETSET program, taking the transverse motion of hadrons into account. The resulting weights were found to be well-behaved and described several manifestations of Bose-Einstein correlations in two-jet events, but the calculation process is rather labourous and is not easy to generalise to include more complex jet topologies.

We propose a significantly simplified method of calculating event weights according to eq. (6), which, in principle, can be applied a posteriori to pre-generated event samples. We suggest that the combination $\kappa \Delta A_{PP'}$ can be estimated as the product of an average interaction radius R and the "relative momentum" Q, which characterises the difference in kinematics between the two permutations P and P'. If the configuration P' is obtained from the configuration P by permuting P identical bosons with masses P and P' and P' is defined as

$$Q^{2} = (p_{1} + \ldots + p_{n})^{2} - n^{2}m^{2}, \tag{7}$$

which coincides with the usual definitions $Q_{12}^2 = -(p_1 - p_2)^2$ and $Q_{123}^2 = -(p_1 - p_2)^2 - (p_1 - p_3)^2 - (p_2 - p_3)^2$ for n = 2 and n = 3, respectively. So, we propose the following replacement:

$$\kappa \Delta A_{PP'} \rightarrow RQ,$$

$$\frac{b}{2} \Delta A_{PP'} \rightarrow \xi RQ,$$
(8)

where ξ is a parameter whose value is to be determined phenomenologically. This leads to the weight calculated as

$$w_P = 1 + \sum_{P' \neq P} \frac{\cos(RQ)}{\cosh(\xi RQ)} \,. \tag{9}$$

For example, in the simplest case of two identical particles, the weight is

$$w_2 = 1 + \frac{\cos(RQ_{12})}{\cosh(\xi RQ_{12})} \,. \tag{10}$$

As noticed in [7], for ξ values around 1 the weight (10), shown as the solid line in Figure 1, is fairly close to the Gaussian-type function

$$w_G = 1 + \exp(-R^2 Q_{12}^2) \tag{11}$$

(dashed line in Figure 1), used as the basic weight in a number of previous studies [6-8]. The important difference is that the new basic weight (10) goes slightly below unity for some intermediate values of Q, while the Gaussian weight (11) always lies above unity. When the

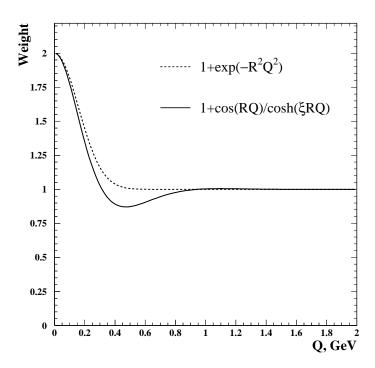


Figure 1: The basic weight (10) used in this paper, compared to the Gaussian basic weight (11), for R = 0.9 fm and $\xi = 1.125$.

weight of an average event is built as a product of many terms of the type (10) or (11), the latter may result in very large event weights, while the former tends to yield event weights close to unity.

Consider, for example, the reaction $e^+e^- \rightarrow W^+W^-$, with both Ws decaying hadronically, and three π^+ mesons in the final state. Let the pions 1 and 2 come from the W⁺ decay while the pion 3 comes from the W⁻. The weight for such an event has the following structure:

$$w_3 = 1 + (12) + (13) + (23) + 2 \times (123),$$
 (12)

where each term stands for one of the six possible permutations. It has the form $\cos(RQ_{(\alpha)})$ / $\cosh(\xi RQ_{(\alpha)})$, where the numbers in brackets, $(\alpha) = (12), (13), \ldots$ show which pions have been permuted. The unity corresponds to no permutation, i.e. initial configuration P. The second term describes the only permutation if no inter-W correlations are allowed, while the following two terms stand for the two 2-particle inter-W permutations. The last term describes two 3-particle permutations (corresponding to the so called "genuine" 3-boson correlations [14]).

In a final state with 4 identical pions (1 and 2 from W⁺, 3 and 4 from W⁻) there are 4! terms:

$$w_4 = 1 + (12) + (34) + (12)(34) + (13) + (14) + (23) + (24) + (13)(24) + (14)(23)$$

$$2 \times [(123) + (124) + (234) + (134)] + 6 \times (1234).$$
(13)

The first line contains only permutations of pions originating from the same Ws, while the remaining terms are either inter-W, or mixed. Note a new term type, e.g. (12)(34), corresponding to the simultaneous permutation within two pairs of pions.

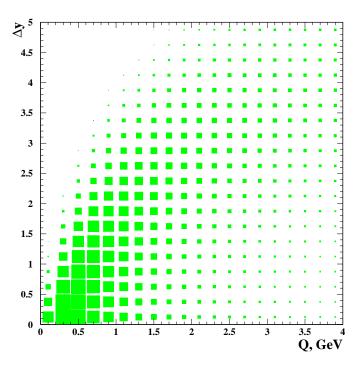


Figure 2: Maximum rapidity difference in a cluster of identical charged pions vs. the variable Q characterising the cluster.

In the hadronic final states produced in high energy collisions, number of identical mesons of each type can be rather big (e.g. 20 or more in hadronic WW events). Of course, only those mesons should be allowed to participate in BEC, which were produced directly during or shortly after the hadronisation phase. In the weight calculation we have included only those mesons whose parents have travelled less than $d_{\rm max}$ in the centre of mass frame. This excludes mesons from the decays of long-lived parents, such as B and D mesons, τ lepton etc., leaving on average about 40% of all mesons. The remaining mesons should be subject to BEC, but a straightforward application of the procedure described above would still lead to serious computational difficulties due to the big number of permutations. However, most of these permuted configurations would have a near-zero contribution to the weight, as their respective values of the "distance" in the momentum space, Q, tend to be high. The weight of each event is essentially determined by clusters of bosons with small values of Q.

In order to eliminate unnecessary calculations, we have ordered all participating mesons of a particular type according to their rapidity $y = (1/2) \ln[(E + p_z)/(E - p_z)]$ (calculated against the thrust axis of the event), and used the strong correlation existing between the value of Q characterising the cluster and the maximum rapidity difference Δy between the mesons in the cluster (see Figure 2). A new cluster was started if the rapidity difference between a meson and the first meson of the current cluster exceeded Δy_{max} . However, no cluster was allowed to contain more than n_{max} mesons. The total weight for a system of mesons of a particular type was calculated as the product of the cluster weights.

Separate weights corresponding to 9 types of mesons $(\pi^+, \pi^0, \pi^-, K^+, K^0, \overline{K}^0, K^-, \eta, \eta')$ were calculated for each event, and the event as a whole was assigned a weight equal to the product of these 9 weights.

3 Choice of parameters

In order to apply the algorithm described above to simulate BEC in Monte Carlo generated events, the values for the parameters R, d_{max} , n_{max} , Δy_{max} and ξ have to be fixed. R essentially describes the effective radius of BEC, which has been measured in various studies at LEP [15–17] to be within 0.5 and 1.0 fm. In the following, unless stated otherwise, we use R = 0.9 fm.

The distance d_{max} should be small enough to exclude from BEC the decay products of long-lived resonances. In our studies we used $d_{\text{max}} = 10$ fm. Note that in our approach the usual parameter λ , which governs the "strength" of BEC, is missing altogether: identical bosons either fully participate in BE correlations (if they are produced early during hadronisation), or do not participate at all (if they come from long-lived parents). So the choice of d_{max} , which determines the fraction of participating bosons, also determines the "effective strength" λ_{eff} , measured by experiments.

The maximum cluster size $n_{\rm max}$ should be chosen large enough to allow 2- and 3-particle correlations measured by experiments, but small enough to keep calculations manageable. It was found that the choice $n_{\rm max}=4$ gives the best overall results, and we have used this value in our calculations. The maximum allowed rapidity difference in a cluster, $\Delta y_{\rm max}$, was chosen to be equal to 6/n, where n is the number of identical bosons of a particular type in an event. This means $\Delta y_{\rm max} \lesssim 1-1.5$, which is fairly harmless, as the clusters with larger Δy typically have rather large Q (see Figure 2) and their contribution to the event weight is small.

The parameter ξ , which is defined by the ratio of the two scales, κ and b, characterising the hadronisation process, determines the value of the argument RQ for which the basic weight becomes smaller than unity. In typical e^+e^- events the Q-distribution of identical meson pairs subject to BEC (see Figure 3) is such that one can find a value of ξ for which the average event weight equals unity. In practice, finding this value of ξ may involve some trial-and-error and interpolation, and is only possible up to a certain precision, determined by Monte Carlo statistics and the variance of the weight distribution. However, once found, it appears to be fairly stable under variation of other parameters.

4 Influence of event weighting on Z properties

As long as the average event weight for Z hadronic decays is equal to unity, the cross section of this process is not changed by event weighting. However, other measurable properties of the Z could be affected.

Since the parameters of the Monte Carlo hadronisation models, which do not explicitly include BEC effects, have been carefully tuned to reproduce various measured distributions, uncritical application of event weights may lead to large inconsistencies with measured partonic branching ratios, different jet topologies, final hadron multiplicities etc. [7]. Also, the average event weight adjusted to unity at one energy may deviate from unity at other energies, thus potentially affecting such parameters as the mass and the width of the Z boson.

In order to study how serious these effects are and to judge what consequences they have for the analysis of the WW events, we have compared various weighted and unweighted distributions. We have used PYTHIA 6.125 Monte Carlo [4] to generate a sample of 10^5 hadronic events at $\sqrt{s} = M_Z = 91.2$ GeV, with ξ adjusted so that the average event weight is equal to 1 (within statistical errors). The distribution of event weights is shown in Figure 4. It peaks close to its average value and is fairly narrow, with an rms of about 0.5.

Table 1 presents average event weights and respective rms values for various initial quark flavours and different jet topologies (as determined by the PYCLUS jet finding algorithm with

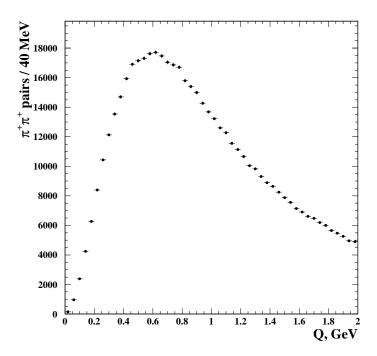


Figure 3: Q-distribution of identical charged pions subject to BEC in Z decays, generated with PYTHIA Monte Carlo (see Section 4).

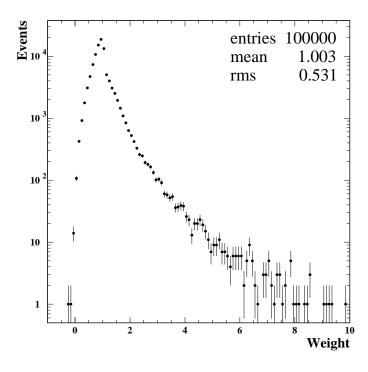


Figure 4: Distribution of event weights in Z decays, for R=0.9 fm and $\xi=1.125$.

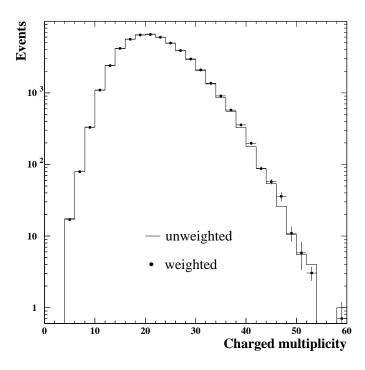


Figure 5: The charged multiplicity distribution for events weighted according to our recipe (data points) compared to the unweighted distribution (solid line). The error bars shown correspond to the errors specific to the event weighting process, and do not include the statistical uncertainties common to both distributions.

default parameters [4]). As seen from the table, the average weights are essentially independent of the initial flavour and number of jets in the event. This means that the implementation of event weights in the form (9) does not cause any noticeable changes in either the partonic branching ratios or the jet activity in Z decays. This is not trivial, as the patterns of the heavy and light quark fragmentation are rather different, and the multiplicity of low-momentum particles is strongly correlated with the number of jets in the event.

Figure 5 shows a very good agreement between the charged particle multiplicity distributions at the Z peak with (data points with errors) and without (solid line) event weighting. Only the errors specific to the weighting process are shown on the plot. The means of the two distributions differ by $\Delta n_{ch} = 0.07 \pm 0.003$, well within the combined experimental error obtained by four LEP experiments, ± 0.11 [18]. Similarly, a very good agreement between the weighted (data points) and unweighted (solid line) momentum distributions of π^+ mesons at the Z peak is shown in Figure 6. The same is true for other hadron types.

In order to study the dependence of the average weight upon initial energy, four more samples of events $e^+e^- \rightarrow \gamma^*/Z^* \rightarrow$ hadrons of the same size were generated at 30, 131, 161 and 200 GeV, using the same value of ξ . Average event weights for these energies are also presented in Table 1. The weights are fairly independent on the initial energy of the collision, although a decrease of about 1% is seen at the highest energy, 200 GeV. This could be connected to the fact that at higher energies a slightly larger percentage of hadronic resonances (such as ρ and K^*) escape the 10 fm limit, which means that their decay products no longer contribute to the weight.

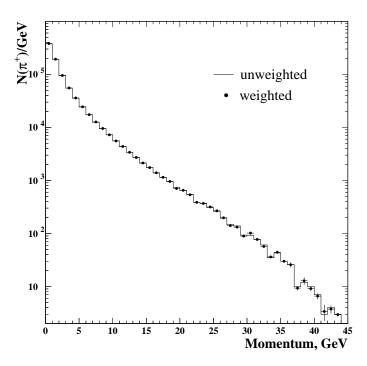


Figure 6: The momentum distribution of π^+ mesons, with events weighted according to our recipe (data points) compared to the unweighted distribution (solid line). As in Fig. 5, the error bars shown correspond to the errors specific to the event weighting process, and do not include the statistical uncertainties common to both distributions.

$qar{q}$	$d ar{d}$	$u\bar{u}$	$sar{s}$	$c\bar{c}$	$b ar{b}$
$\langle w \rangle$	1.016 ± 0.004	1.010 ± 0.005	1.000 ± 0.004	1.002 ± 0.004	0.991 ± 0.003
rms	0.573	0.595	0.520	0.482	0.463
$N_{ m jet}$	2	3	4	5	6
$\langle w \rangle$	1.009 ± 0.003	0.997 ± 0.002	1.000 ± 0.003	1.015 ± 0.007	1.068 ± 0.023
rms	0.482	0.429	0.559	0.713	1.053
$E_{\rm cm}$	30 GeV	91.2 GeV	$131 \mathrm{GeV}$	$161 \; \mathrm{GeV}$	200 GeV
$\langle w \rangle$	0.995 ± 0.001	1.003 ± 0.002	0.994 ± 0.002	0.991 ± 0.002	0.988 ± 0.002
rms	0.343	0.531	0.526	0.503	0.494

Table 1: Average weights, $\langle w \rangle$, and rms values for different quark flavours, jet topologies and centre of mass energies.

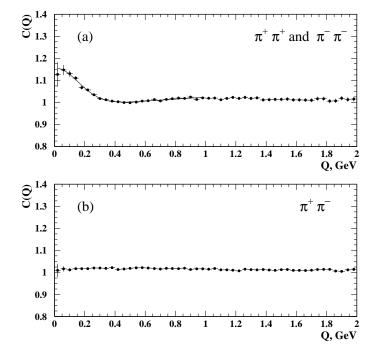


Figure 7: The correlation function C(Q), calculated as the ratio of weighted and unweighted Q distributions, for same-sign charged pions (a) and opposite-sign charged pions (b). The line in (a) shows the fit result using the parametrisation (14), with parameter values given in Table 2 (event weight).

In order to study possible effects of the event weighting upon the mass and the width of the Z resonance, four more samples of 10^5 events were generated at $\sqrt{s} = M_{\rm Z} \pm 2$ GeV and $M_{\rm Z} \pm 4$ GeV, with the same value of the parameter ξ . A Breit-Wigner fit to these five points with and without event weighting yielded no significant shifts in the Z mass and width: $\Delta M_{\rm Z} = 0.4 \pm 0.5$ MeV and $\Delta \Gamma_{\rm Z} = 1.7 \pm 1.7$ MeV. The experimental uncertainties on these extremely precisely measured quantities are 2.2 MeV and 2.6 MeV, respectively [18].

The correlation functions C(Q) were constructed as ratios of the Q distributions of particle pairs for weighted and unweighted events. The ratios for same- and opposite-sign charged pion pairs are shown in Figure 7. A clear enhancement is seen for the same-sign pion pairs at small values of Q, while the distribution for the opposite-sign pairs is flat and close to 1. Also shown is the result of the fit to the same-sign pair correlation function of the form

$$C(Q) = N(1 + \beta Q)(1 + \lambda_{\text{eff}} \exp(-Q^2 R_{\text{eff}}^2)),$$
 (14)

which is often used to parametrise the experimentally observed correlation function in Z decays. The values obtained for the parameters (for a fit range of 0–1 GeV in Q) are presented in Table 2, together with the results of a similar fit to the input pair weight (the latter is merely a fit of the form (14) to the basic weight described by (10)). The statistical errors in the fitted Q distributions were increased by 40%, in order to account for the bin-to-bin correlations, arising from the fact that each boson in an event can contribute to several combinations.

Fit results for input (pair weight) and output (event weight) parameters are in a reasonable agreement with each other, the only noticeable difference being that the observed enhancement

	Event weight	Pair weight
$\lambda_{ ext{eff}}$	0.181 ± 0.012	0.186 ± 0.011
N	0.978 ± 0.008	0.951 ± 0.009
$\beta \; (\mathrm{GeV^{-1}})$	0.046 ± 0.013	0.053 ± 0.013
$R_{\rm eff} \ ({\rm fm})$	0.902 ± 0.056	0.758 ± 0.038

Table 2: The fitted values of the output (event weight) and input (pair weight) correlation function parameters at the Z peak.

at small Q is slightly narrower than in the input distribution, leading to a larger effective value of R_{eff} .

So, the event weighting method described above reproduces the BE correlation functions without introducing any significant modification of the properties of the Z boson, and no noticeable energy dependence of the average weight, using the same value for the single adjustable parameter ξ for all types of events across the whole energy range considered.

We have repeated most of our studies for R=0.6 fm with very similar results. The average event weight was equal to unity (within statistical errors) for $\xi=0.98$, with no significant dependence on energy, initial quark flavour or number of jets. The fitted values for the parameters of the correlation function (14) were: $\lambda_{\rm eff}=0.29\pm0.03$ and $R_{\rm eff}=0.61\pm0.04$. The values of these parameters, obtained by LEP experiments studying Z hadronic decays, vary in the intervals 0.2–0.6 for $\lambda_{\rm eff}$ and 0.5–1.0 fm for $R_{\rm eff}$, depending on the analysis [15–17]. More meaningful and detailed comparisons of our results with real data will only be possible when the real-world analyses are applied to the generator-level Monte Carlo samples, weighted according to our recipes.

5 BEC in W pair production

We have studied possible influence of inter-W BE effects on the apparent mass and other measured properties of the W boson. The PYTHIA 6.125 event generator [4] was again used to simulate the process $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$, and the weighting method described above was applied to implement Bose-Einstein effects. For obvious reasons, final-state meson distributions in this process are quite different from those in Z decays considered above, and one should expect to obtain the average weight equal to unity for a different value of the parameter ξ .

In weight calculations we have used the same source radius as for Z studies, R=0.9 fm. Three different weighting schemes were considered:

- Only identical bosons originating from different Ws were included in Bose-Einstein correlations (labelled as DW scheme).
- Only bosons from the same W are subject to BEC (labelled as SW scheme).
- All identical bosons from both the same and different Ws are allowed to participate in Bose-Einstein effects (labelled as SW+DW).

Two samples of 25000 events were generated at the energies 161 and 200 GeV. We have tuned ξ to obtain the average value of the event weight approximately equal to unity at the energy 161 GeV in each of the three schemes separately, and then used the same values of ξ at

	ξ	$\langle w \rangle$	rms	ΔM_W	$\Delta\Gamma_W$	Δn_{ch}
				(MeV)	(MeV)	
161 GeV						
DW	1.008	0.989 ± 0.003	0.491	13 ± 5	-11 ± 18	0.12 ± 0.005
SW	1.048	1.007 ± 0.004	0.679	-2 ± 6	-45 ± 26	0.19 ± 0.005
SW+DW	1.094	1.002 ± 0.006	0.930	11 ± 9	-63 ± 33	0.27 ± 0.005
(SW+DW)-(SW)				13 ± 6	-18 ± 20	0.08 ± 0.005
200 GeV						
DW	1.008	0.983 ± 0.003	0.430	1 ± 4	3 ± 11	0.08 ± 0.005
SW	1.048	0.986 ± 0.004	0.623	-5 ± 5	-34 ± 15	0.11 ± 0.005
SW+DW	1.094	0.986 ± 0.005	0.814	-7 ± 6	-37 ± 20	0.17 ± 0.005
(SW+DW)-(SW)				-2 ± 4	-3 ± 13	0.06 ± 0.005

Table 3: Parameter ξ , average weights and their rms for W⁺W⁻ hadronic decay events at 161 and 200 GeV, together with shifts in W mass, width and charged multiplicity caused by event weighting.

200 GeV. The application of the parameter ξ , fixed at 161 GeV, to the higher energy, 200 GeV, reduces average event weights by about 1.5%, apparently because at higher energies, due to the boost of the W decay frame, more resonances travel beyond the allowed limit d_{max} . A similar reduction of the average weight was observed when we applied our algorithm to semileptonic WW events, $e^+e^- \to W^+W^- \to q\bar{q}l\nu$ at 200 GeV, using the value of ξ adjusted for Z decays.

Table 3 presents the values of ξ , average event weights and their rms, together with the shifts in the mass and the width of the W and the average charged particle multiplicity in hadronic W decays, compared to the case with no event weighting. The mass and the width were determined by fitting a Breit-Wigner parametrisation to the invariant mass distribution of the W decay products.

The fact that the shifts in W parameters for the SW scheme differ from zero shows that our implementation of Bose-Einstein effects is not perfect, as we do not know any valid reason why the inclusion of BEC only for bosons originating from same W should change any of them (see a similar discussion on Z properties in Section 4). However, we expect that the differences in these quantities between SW+DW and SW represent a valid estimate of the effects of inter-W BE correlations, alongside with the predictions of the DW scenario. Note that these two sets of shifts (the first and the last rows for each energy in Table 3) are consistently close to each other. Thus, averaging over these two scenarios, our simulations show that the shifts in the W mass, width and average charged multiplicity in W decays respectively are 13 ± 5 MeV, -15 ± 18 MeV and 0.10 ± 0.005 at 161 GeV, reducing correspondingly to 0 ± 4 MeV, 0 ± 11 MeV and 0.07 ± 0.005 at 200 GeV. Typical experimental errors on these quantities are at present significantly larger: 56 MeV, 50 MeV and 0.4 [18].

Table 4 presents values of the average event weight and the rms of the event weight distribution, for varous jet topologies in the DW scenario. The average weights are essentially independent of the number of jets. Hence, the implementation of event weights does not change significantly the jet multiplicity distribution in the W⁺W⁻ production process. The rms of the weight distribution increases slightly with the number of jets, as in Z decays. This is connected to the increase of particle multiplicity with increasing number of jets. The event weight dependence on the flavours of the initial quarks in W decays is very weak and does not change partonic branching ratios. We have also checked that the introduction of event weights

$N_{ m jet}$	4	5	6	7	8
161 GeV					
$\langle w \rangle$	0.964 ± 0.008	0.969 ± 0.005	0.982 ± 0.005	1.001 ± 0.007	1.030 ± 0.012
rms	0.352	0.410	0.443	0.558	0.641
200 GeV					
$\langle w \rangle$	0.968 ± 0.009	0.970 ± 0.005	0.978 ± 0.005	0.984 ± 0.005	1.000 ± 0.008
rms	0.319	0.344	0.393	0.430	0.483

Table 4: Average event weights and the rms for different number of jets at 161 and 200 GeV, in the DW scheme.

	DW	SW	SW+DW
$161 \; \mathrm{GeV}$			
$\lambda_{ ext{eff}}$	$0.073 \pm .012$	$0.117 \pm .013$	$0.201 \pm .015$
$R_{\rm eff}$ (fm)	$1.026 \pm .126$	$1.006 \pm .086$	$1.053 \pm .057$
$200~{\rm GeV}$			
$\lambda_{ ext{eff}}$	$0.057 \pm .010$	$0.116 \pm .012$	$0.182 \pm .014$
$R_{\rm eff}$ (fm)	$0.902 \pm .107$	$0.994 \pm .081$	$0.993 \pm .055$

Table 5: The fitted values of the correlation function parameters λ_{eff} and R_{eff} for W pair production in (DW), (SW) and (SW+DW) schemes at $\sqrt{s} = 161$ and 200 GeV.

does not alter multiplicity distributions and inclusive momentum spectra for various hadrons in W⁺W⁻ production. Similar results were obtained also for SW+DW and SW event weighting schemes.

On the experimental side, it is still not clear at the moment whether the Bose-Einstein correlations between mesons originating from different Ws exist or not [19–22]. Therefore, we have studied the correlation functions for identical bosons in all three scenarios (DW, SW and SW+DW). The correlation functions were constructed as ratios of particle pair four-momentum difference distributions for weighted and unweighted W^+W^- hadronic decay events. They are plotted in Figure 8 for the charged pion case. In all three scanarios, the figure shows a clear enhancement at small Q, characteristic of Bose-Einstein correlations.

Values for the effective strength of correlations $\lambda_{\rm eff}$ and the observed effective source radius $R_{\rm eff}$, obtained by fitting the form (14) to these distributions, are given in Table 5. The strength of the correlations in the DW case is significantly smaller compared to the full SW+DW scenario, because a large number of identical bosons come from the same W, and the pairs of identical bosons from different Ws are on average farther away from each other, in the Q space, than those from the same Ws. Hence, the experimental observation of Bose-Einstein correlations for charged pion pairs originating from different Ws at LEP2 would indeed be very difficult, requiring high statistics, and a careful control of systematics coming from the choice of the reference sample. As in the case of hadronic Z decays, the effective input (pair weight) and output (event weight) values for parameters $\lambda_{\rm eff}$ and $R_{\rm eff}$ are reasonably close to each other.

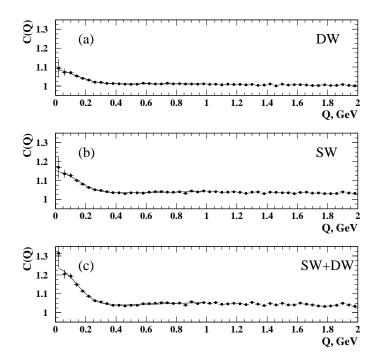


Figure 8: The correlation functions C(Q), for the three different scenarios described in the text: a) only bosons from different Ws participate in BEC (DW); b) only bosons from the same W participate in BEC (SW); c) all identical bosons participate in BEC (SW+DW). The lines show the fit results using the parametrisation (14), with parameter values given in Table 5.

6 Conclusions

We have developed a method for modelling Bose-Einstein correlations using event weighting, with a theoretically motivated parametrisation for the basic weight, based on the string fragmentation picture. Our approach differs from some other "global weighting" schemes, as the event weights in our case are distributed both above and below unity with a rather narrow distribution, and the average event weight is easily adjusted to unity using a single parameter ξ . This eliminates the additional rescaling of event weights, necessary in models where all event weights are larger than one.

By weighting Monte Carlo events in accordance with our prescriptions, the experimentally observed characteristic Bose-Einstein enhancement at small relative momenta is reproduced. Good agreement was found between the input and output values of the parameters $\lambda_{\rm eff}$ and $R_{\rm eff}$ in the de-facto standard Gaussian parametrisation. By fine-tuning the values of our model parameters $d_{\rm max}$ and R (and subsequent re-adjustment of ξ in order to keep the average weight equal to unity) one can bring $\lambda_{\rm eff}$ and $R_{\rm eff}$ closer to the values obtained by particular experiments. However, any detailed comparison of our results with the real data will only be possible when the real-world analyses are applied to the Monte Carlo samples, weighted according to our recipes.

The main weakness of all BEC implementations via event reweighting is the possible violation of factorization between the hard perturbative part of the process and the non-perturbative hadronisation stage, essential to all Monte Carlo generators. Our model is practically free from these difficulties. We have made extensive checks by comparing our predictions with unweighted distributions, which have been tuned and tested to reproduce very precise experimental data on Z decays. We have found no significant shifts in the mass, width and partonic branching fractions of the Z boson due to event reweighting, within the estimated errors which are well below the level of existing experimental uncertainties. The same is true for charged multiplicity distributions and inclusive spectra of various final-state hadrons.

In the process $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$, the introduction of event weights leads to small shifts in the values of W mass, width and charged multiplicity, less than 15 MeV, 20 MeV and 0.10, respectively at $\sqrt{s}=161$ GeV, and even smaller at 200 GeV. These values are well below currently existing experimental errors. Hence, at the generator level the Bose-Einstein correlations as implemented in our model do not introduce large additional uncertainties in the determination of W characteristics in the fully hadronic channel. However, as in the case of Z decays, in order to assess the effects of BEC on the experimentally measured W parameters, the detector simulation and the actual fitting procedures used by the LEP experiments have to be applied to the weighted event samples. This is not too difficult because there is no need to generate special Monte Carlo samples, as the weighting can be applied a posteriori to the existing Monte Carlo events.

We have also studied the correlation functions for charged pions in fully hadronic WW events in three separate scenarios, depending on which pairs of identical bosons were allowed to participate in BEC: only from different Ws (DW), only from the same W (SW) and all pairs (SW+DW). The characteristic enhancement at $Q \leq 0.2$ GeV was seen in all three scenarios. However, the effective value of the parameter $\lambda_{\rm eff}$ is the smallest in the (DW) scenario, suggesting that the direct observation of BEC for pions originating from different Ws with the available statistics from LEP2 would be difficult, requiring a very careful control of systematics.

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